

The cosmological constant as a residual energy in the chaotic inflationary model

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Abstract:

A new idea of the cosmological constant is proposed in this paper. Due to the horizon is limited, the quantum fluctuation of the inflaton field is not zero, a nonzero vacuum energy is remained as a residual inflationary energy of an unusual potential, however the true stable vacuum energy is zero fortunately. A unified model of the cosmological constant and the chaotic inflation is proposed, which satisfies almost all cosmological phenomenology and will can be tested by data of the cosmic large scale structure.

1 Introduction

An astonished result of the high redshift type Ia supernovae observations in 1998 is that the deceleration parameter comes out negative^[1,2], implying that our universe is speeding up. The current analyses favor $\Omega_m \simeq 0.4$ and $\Omega_\Lambda \simeq 0.6$, and the universe is flat. The accelerated expansion of the universe may be induced by a nonzero cosmological constant, which corresponds to $\Lambda_0 = 3M_p^2 H_0^2$, $\Omega_\Lambda \simeq (2\text{meV})^4$, where $\tilde{M}_p = \sqrt{8\pi}M_p \simeq 1.2 \times 10^{19}\text{GeV}$ is the Planck energy scale, and the present Hubble constant is $H_0 = 100h \text{ km/sec/Mpc} \simeq 5.6 \times 10^{-61} M_p$ if we take $h \simeq 0.65$.

The relatively tiny value for a cosmologically relevant Λ_0 seems very fine tuned from the particle physics point of view and so the prejudice in a large part of the particle physics community has been that it should be zero, by some mechanism not yet understood^[3-5]. An extremely small but nonzero Λ_0 could perhaps be taken as some small effect perturbing around the preferred value of zero. A lot of new ideas have been suggested, such as rolling scalar field^[6], variable cosmological constant, X-matter, generalized dark matter, loitering universe, quintessence model, tracker field and attractor solutions, tangled strings^[7] and textures^[8], and so on. However, one should still continue to seek a better explanation on the essence of the nonzero and tiny cosmological constant. The purpose of this letter is to suggest a new model

in which the chaotic inflation and the cosmological constant are unified. In spite of the true theoretical cosmological constant is zero, the observing cosmological constant as a relic energy of the inflation potential is nonzero actually due to the finite horizon and the quantum fluctuation of the inflaton field.

2 The universality of the quantum fluctuation of the inflaton field

We note that the cosmological constant, i.e., vacuum energy, has been playing an important role in the inflationary epoch of the early universe. This unstable vacuum energy is very large during that time. Our first question is whether the large cosmological constant of the inflation is same with the present tiny cosmological constant in essence? We think so and suppose that both can be unified in a common inflationary potential, i.e., the essence of both is same. The second question is what kind of potentials should be adopted by us? We suppose that the lowest stable vacuum must site at a point of $\phi = 0$ and $V(\phi) = 0$, i.e., the true vacuum must have a zero cosmological constant from the view point of the naturalness. The reason that the position of the lowest stable vacuum is taken at $\phi = 0$ by us is that we worry that a nonzero large average value of the true vacuum $\langle \phi \rangle \neq 0$ may give easily some particles with dangerous large masses due to the singlet property of the inflaton ϕ . In according to an inference of the inflation theory, after the slow rolling of the inflation the universe will rapidly roll down to its true vacuum state, i.e., $V(0) = 0$ point, thus there is not any nonzero cosmological constant to be remained. However, this inference may be wrong. Since many reasons hint that the inflation field ϕ may have a nonzero uncertainty value $\delta\phi$, thus the actual vacuum energy $V(\delta\phi)$ will be nonzero.

In fact the inflaton field ϕ disassemble into two parts during its evolutionary process, the unstable vacuum average value $\langle \phi \rangle$ and the quantum fluctuation $\delta\phi$, i.e., $\phi = \langle \phi \rangle + \delta\phi$. The quantum fluctuation of de-Sitter universe is determined by its expansion rate $H = \dot{R}/R$, i.e., $\delta\phi = H/(2\pi)^{[9]}$, which is often used in the analysis of the fluctuation spectrum of the inflationary universe. We shall made an important supposition which says that the relation $\delta\phi = H/(2\pi)$ should be suitable for all cases of expanding universe, not merely for de-Sitter one. We can deem that this assumption is reasonable due to the following four reasons. First, when we apply the relation $\delta\phi = H/(2\pi)$ in the case of the early inflation universe, the universe in this time is actually not an exact de Sitter one, but we can use it successfully. Second, for each moment the expanding universe can be taken approximately as a de-Sitter universe with the expansion rate H . Third, the age of the universe is inverse proportional to the Hubble constant H , the uncertainty of the energy is proportional to H , the relation $\delta\phi = H/(2\pi)$ seems to coincide with the uncertainty principle of the quantum mechanism. Four, in according to the holographic hypothesis of the universe^[10], some property of the bulk of the universe is determined by its boundary,

i.e., the horizon H .

In spite of $\langle \phi \rangle$ is much larger than $\delta\phi$ during the period of the inflation due to slowly rolling, however in contrast, now the quantum fluctuation $\delta\phi$ which is not controlled by rolling process will be much larger than the value $\langle \phi \rangle$ for the present universe. Due to nonzero value of $\delta\phi = H/(2\pi)$, the present universe has a nonzero cosmological constant $V(H/(2\pi))$. This is just our new understanding to the essence of the actual nonzero cosmological constant, i.e., it is a relic energy of the inflation potential.

3. An unusual effective potential of the inflaton field

Due to the complications of generating the effective inflation potential, we do not know its exact form at present. These complexities include that the high loop quantum modifications, a running of the parameters, the compactification and evolution of the internal dimensions, and the various non-perturbative effects from the quantum field theory, quantum gravity, superstring and even the M-theory. No matter how complicated of its generating, we can always adopt an effective potential to describe it, even though which form looks like very strange.

We take the following effective potential for whole evolution of the universe:

$$V(\phi) = m_{(\alpha)}^{4-\alpha} |\phi|^\alpha + m_{(\beta)}^{4-\beta} |\phi|^\beta,$$

where the first term is responsible for the cosmological constant, the second term is responsible for the chaotic inflation. We suppose that the parameters $0 < \alpha \lesssim 2/3$, $\beta \simeq 1.9$ and $m_{(\alpha)} \ll m_{(\beta)}$, the reason will be given later on. We shall demonstrate that this potential satisfies almost all requirements of the cosmological phenomenology. The lowest point of this potential is indeed $V(0) = 0$, satisfying the naturalness condition suggested by us.

4. Inflation and preheating

In according to the inflation theory, the slowly rolling parameters are

$$\varepsilon = \frac{M_p^2}{2} \left(\frac{V'}{V} \right)^2 \simeq \frac{\gamma^2 M_p^2}{2\phi^2}, \quad \eta = M_p^2 \frac{V''}{V} \simeq \frac{\gamma(\gamma-1)M_p^2}{\phi^2},$$

where the parameter $\gamma = \alpha$ for $\phi \ll m_{(\alpha)}$ and $\gamma = \beta$ for $\phi \gg m_{(\beta)}$. The inflation should end at $\phi_{\text{end}} \simeq 0.3M_p$ where $\varepsilon_{\text{end}} - \eta_{\text{end}} = \beta(2-\beta)\phi_{\text{end}}^{-2}M_p^2/2 \simeq 1$. The parameter β is not equal to 2 in order to avoid $\varepsilon - \eta$ being zero, otherwise the universe can not end its inflation. The e-fold of the inflation is

$$N = \int \frac{V d\phi}{M_p^2 V'} \simeq \frac{\phi_{\text{begin}}^2 - \phi_{\text{end}}^2}{2\beta M_p^2}.$$

In order for our universe to have an enough inflating e-fold $e^N \gtrsim 10^{61}$, our universe must begin to inflation at $\phi_{\text{begin}} \gtrsim 23M_p$. Before ϕ_{begin} , the universe undergoes the quantum production^[11] and the chaotic inflation^[12]. The density fluctuation of the universe is given by^[13]

$$\delta^2 = \frac{V}{150\pi^2 M_p^4 \epsilon} \simeq \frac{m_{(\beta)}^{4-\beta} \phi^{\beta+2}}{75\pi^2 M_p^6 \beta^2},$$

which must be about an order magnitude of 10^{-10} , then we obtain $m_{(1.9)} \simeq 2.2 \times 10^{-6} M_p = 5.3 \times 10^{12} \text{GeV}$, which is far below the grand unification energy scale. The spectrum index $n = -6\epsilon + 2\eta \simeq 0.014$, the spectrum is almost scale invariant.

After the end of the slowly rolling, the inflaton field begins its fast oscillation, which effective mass is $m_\phi^2 = V'' \simeq \beta(\beta-1)m_{(\beta)}^{4-\beta} \phi^{\beta-2}$. In the moment of inflation end, the effective mass of inflaton is $m_{\phi \text{ end}} \simeq 0.7m_{(1.9)}$. The unstable vacuum energy $V_{\text{end}} = m_{(1.9)}^{2.1} (\phi_{\text{end}})^{1.9} \simeq (6 \times 10^{-4} M_p)^4$ transforms into a lot of the zero temperature inflaton particles. When the inflaton field rolls down, the field average value of inflaton reduces, however the effective mass of the inflaton increases (if $\beta < 2$) and becomes larger than the mass of fermion, which has a coupling $h\bar{\psi}\psi\phi$ with the inflaton, then all inflatons begin to decay into pairs of the fermions. This turning point is determined by equation $h^2 \phi_{\text{turn}}^2 = \beta(\beta-1)m_{(\beta)}^{4-\beta} \phi_{\text{turn}}^{\beta-2}$. In this case $m_{\phi \text{ turn}} \simeq 1.3m_{(1.9)}$ if the Yukawa coupling constant is $h \simeq 1$. The decay width is $\Gamma = h^2 m_{(\beta)} / (8\pi) \simeq 2.7 \times 10^{11} \text{GeV}$, which can be viewed as the preheating temperature T_{preh} . This T_{preh} is not only enough low to avoid to produce harmful topological defects, but also enough high to create some super heavy long lifetime particles, which can just generate the highest energy cosmic rays in our universe. If $\beta > 2$, the inflaton will decay too early to avoid producing dangerous topological defects. The fermion come from the decay of the inflaton may be the gaugino, which can easily produce all elementary particles of the supersymmetric standard model in a thermal universe.

5 5.The cosmological constant as a residual inflationary energy

When the inflaton field continue to roll down rapidly, the part of the average value $\langle \phi \rangle$ disappears, and bequeaths the part of the quantum fluctuation $\delta\phi = H/(2\pi)$ as its value, which is very small. Therefore the potential leaves a nonzero cosmological constant Λ_0 ,

$$m_{(\alpha)}^{4-\alpha} \left(\frac{H_0}{2\pi}\right)^\alpha = 3M_p^2 H_0^2 \Omega_\Lambda,$$

we must have $m_{(\alpha)} = (3(2\pi)^\alpha \Omega_\Lambda H_0^{2-\alpha})^{1/(4-\alpha)} M_p$, which magnitude order depends on the index α sensitively. We list some data in following: $m_{(1)} \simeq 46 \text{MeV}$ with

$w = -1/2$, $m_{(2/3)} \simeq 3.4\text{keV}$ with $w = -2/3$, $m_{(1/2)} \simeq 55\text{eV}$ with $w = -3/4$, where $w \equiv p/\rho = \alpha/2 - 1$ is a ratio of pressure over density, this formula will given later on. If $\alpha = 0$, we have $m_{(0)} \simeq 2\text{meV}$ with $w = -1$, i.e., true cosmological constant, however in this case $V(0) \neq 0$ which does not satisfy our requirement. This show that the cosmological constant is a relic energy of the inflation potential. The parameter $\beta > 1.8$ will guarantee that the second term is far smaller than the first term of the inflaton potential when the inflaton field value $\delta\phi$ is taken as $H/(2\pi)$.

During the evolution of our universe, the Friedmann equation is^[14]

$$\rho_m + \Lambda = 3M_p^2 H^2.$$

Considering the fluctuation energy of the inflaton field introduced by us, the above equation becomes

$$\frac{\rho_0}{R^3} + \frac{m_\alpha^{4-\alpha}}{(2\pi)^\alpha} \left(\frac{\dot{R}}{R} \right)^\alpha = 3M_p^2 \left(\frac{\dot{R}}{R} \right)^2.$$

This will affect the formation of the large scale structure of our universe and the relevant observation data can be used to test our model! At a period which redshift z is between about 100 and 10, the inflaton energy term can be omitted, the solution is approximately $R \propto t^{2/3}$, the matter density term is $\rho_m \propto t^{-2}$, and the inflaton energy term is $V_{\text{vac}} \propto t^{-\alpha}$, which is equivalent to the quintessence with $w = \alpha/2 - 1$. The observation allow that $w < -2/3$, therefore requiring to $\alpha < 2/3$. If the observing data permits $w \simeq -1/3$ ^[15], we can combine the two term of our potential into a single term with $\alpha = \beta = 4/3$, i.e., the inflation and the cosmological constant are uniformly described by only one term.

Actually our V_{vac} is not the quintessence by its self meaning. As the universe continue to evolution, the difference between both becomes very distinct. When time goes to infinite, $\Omega_\Lambda(\infty) \rightarrow 1.0$. The vacuum energy will reduce, and the Hubble constant will arrive a limit value

$$H_\infty = \left(\frac{\Omega_\Lambda(t_0)}{\Omega_\Lambda(\infty)} \right)^{1/(2-\alpha)} H_0,$$

however if the cosmological constant is a true constant (no variable with time, $\alpha = 0$), the limited values H_∞ will increase.

6 6.A simple remark

In conclusion, we see that a simple potential model including four parameters: two energy scales ($m_{(\alpha)}$, $m_{(\beta)}$) and two power indices (α , β), can explain almost all cosmological phenomenology, from the inflation, slowly rolling, an enough e-fold, a suitable density fluctuation, the flat spectrum index, preheating, avoiding topological defects, explain the highest energy cosmic ray, to a nonzero cosmological

constant in spite of the true one being zero, and its special evolution and affecting on the formation of the large scale structure in our universe. Of course this model has an ability to include some cool dark matters, such as WIMP. However, we face still on an intractable problem why the universe chooses so strange inflation potential? How to calculate the compactification of the extra-dimensions and the non-perturbation quantum effects?

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